Proof of frequency mapping function

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The frequency warping is defined by

$$\hat{z}^{-1} = \frac{z^{-1} - \alpha}{1 - \alpha z^{-1}}$$

substitute $z = e^{j\omega}$, mul and div by conjugate of denominator and the RHS becomes

$$\hat{z}^{-1} = \frac{e^{-j\omega} - \alpha}{1 - \alpha e^{-j\omega}} = \frac{e^{-j\omega} - \alpha}{1 - \alpha e^{-j\omega}} \times \frac{1 - \alpha e^{j\omega}}{1 - \alpha e^{j\omega}} = C \times (\alpha^2 * e^{j\omega} - 2\alpha + e^{-j\omega})$$

$$\hat{z}^{-1} = e^{-j\hat{\omega}} = C \times (\alpha^2 cos(\omega) + j\alpha^2 sin(\omega) - 2\alpha + cos(\omega) - jsin(\omega)) = e^{tan^{-1}\frac{(\alpha^2-1)sin(\omega)}{(1+\alpha^2)cos(\omega)-2\alpha}}$$

$$\hat{\omega} = tan^{-1}\left\{\frac{(\alpha^2 - 1)sin(\omega)}{(1 + \alpha^2)cos(\omega) - 2\alpha}\right\}$$

where C is real constant and doesnot affect the phase component.